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Mixing of two-level unstable systems

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Abstract

Unstable particles can be consistently described in the framework of quantum field theory. Starting from the full S-matrix amplitudes of $B^+ \to (2\pi,\ 3\pi)l^+\nu$ decays as examples in the energy region where the $\rho-\omega$ resonances are dominating, we propose a prescription for the mixing of two quasi 'physical' unstable states that differs from the one obtained from the diagonalization of the $M-i\Gamma/2$ non-hermitian hamiltonian. We discuss some important consequences for CP violation in the $K_L - K_S$ system.

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The $\rho-\omega$ and K_L-K_S mesons are two beautiful examples of two-level mixed systems useful to study important properties of quantum mechanics and fundamental interactions of unstable particles. The effects of isospin breaking in the case of $\rho-\omega$ system and CP violation in the case of K_L-K_S system convert the corresponding eigenstates (ρ^I,ω^I) or (K_1,K_2) into physical eigenstates (ρ,ω) and (K_L,K_S). These systems allow to study the violation of fundamental symmetries where the effects of unstabilities play an essential role.

Unstable particles can be consistently treated only in the framework of quantum field theory[1, 2]. They can not be described by asymptotic states entering the calculation of physical S-matrix amplitudes. Instead, they are associated to propagation amplitudes (propagators) between their production and decay locations and can not be detached from these mechanisms in order to extract truncated amplitudes. In quantum field theory, unstable states or resonances are special cases of non-perturbative phenomena obtained from a full resummation of perturbative bubble graphs [1, 2]. In addition, the space-time behaviour of the amplitudes for production and decay of resonances obey, in extremely good approximation, the celebrated exponential decay law and the covariance properties for the time-evolution amplitudes [2].

The conventional quantum mechanical treatment of symmetry breaking in two-level unstable systems consists in finding the eigenstates that diagonalize a non-hermitian effective hamiltonian of the form $H=M-i\Gamma/2$ [3, 4], where M and Γ are 2×2 hermitian matrices describing the mass and decay properties [5] of the unstable states. H governs the time evolution of the so-called physical eigenstates which at initial time are given by

$$|X\rangle = |X^s\rangle + \epsilon|Y^s\rangle \ , \tag{1}$$

$$|Y\rangle = |Y^s\rangle - \epsilon|X^s\rangle \ , \tag{2}$$

$$\epsilon = \frac{\langle X^s|H^{SB}|Y^s\rangle}{m_X - m_Y + \frac{i}{2}(\Gamma_Y - \Gamma_X)} \ . \tag{3}$$

Here $|Z^s\rangle$ denotes an interaction eigenstate, m_Z (Γ_Z) is the mass (decay width) of the unstable state and ϵ is the mixing parameter due to symmetry breaking. H^{SB} is the symmetry breaking hamiltonian that mixes the X^s and Y^s states. As it could be easily checked, the physical states are non-orthogonal which can be traced back to the non-hermitian character of the hamiltonian.

The purpose of this paper is to demonstrate that the calculation of the full S-matrix amplitude for a process involving the production and decay of mixed resonances, leads to a different mixing prescription for the unstable quasi 'physical states' than the one obtained from the diagonalization of the effective $M-i\Gamma/2$ hamiltonian. In other words, the inclusion of symmetry breaking in the evaluation of transition amplitudes involving the approximation where resonances are described by asymptotic states can be properly done by using the quasi 'physical states' as given below in Eqs. (4)–(5) and not in Eqs. (1) and (2). The numerical impact of using both approaches in the evaluation of symmetry breaking when extracting truncated physical observables as branching fractions for $B \to Vl\nu$ [6, 7], can be very important.

To be more specific let us consider the S-matrix amplitudes of the *full* decay processes $B^+ \to (2\pi, 3\pi)l^+\nu_l$, which are dominated by the intermediate ρ and ω resonances (this example illustrates the main characteristics of a two-level unstable mixed system). We show that a convenient prescription for the physical quantum mechanical eigenstates should be taken as [7]:

$$|\rho\rangle = |\rho^I\rangle + \epsilon'|\omega^I\rangle \ , \tag{4}$$

$$|\omega\rangle = |\omega^I\rangle + \epsilon''|\rho^I\rangle \ , \tag{5}$$

in order to evaluate the matrix elements of the truncated processes $B^+ \to (\rho^0,\ \omega)l^+\nu_l$ in presence of isospin symmetry breaking. In the above Eqs. ϵ' and ϵ'' are given by

$$\epsilon' = \frac{m^2_{\rho\omega}}{m^2_\rho - m^2_\omega + im_\omega\Gamma_\omega} \ , \tag{6}$$

$$\epsilon'' = \frac{m^2_{\rho\omega}}{m^2_\omega - m^2_\rho + im_\rho\Gamma_\rho} \ , \tag{7}$$

where $m^2_{\rho\omega} \equiv \langle\omega^I|H^{\Delta I=1}|\rho^I\rangle$ is the $\rho-\omega$ mixing strength. This results into sizable numerical differences with respect to Eqs. (1–3) in the evaluation of isospin symmetry breaking effects as discussed in Refs. [7].

Let us consider the full S-matrix amplitude for the semileptonic process $B^+(p_B) \to \pi^+(p_1)\pi^-(p_2)l^+(p)\nu_l(p')$, where p_i denotes the corresponding four-momenta (the results for the $3\pi l\nu_l$ decay mode are straightforward). Including the contributions of intermediate isospin eigenstates ($\rho^I,\ \omega^I$) and isospin breaking effects through $\rho-\omega$ mixing [8], we obtain (we assume that only the ρ^I can couple to the $\pi\pi$ system, *i.e.* we ignore a possible *direct* contribution $\omega^I \to \pi^+\pi^-$):

$$\begin{aligned}\mathcal{M}(B \to 2\pi l\nu) \;=\; & \frac{G_F V_{ub}}{\sqrt{2}} l^\mu \left\{ \mathcal{M}_{\mu\alpha}(B^+ \to \rho^{I*})(\mathcal{P}_\rho)^{\alpha\beta}(q) \right. \\ & \left. + \mathcal{M}_{\mu\alpha}(B^+ \to \omega^{I*})(\mathcal{P}_\omega)^\alpha_\nu(q) \cdot i m^2_{\rho\omega} \cdot (\mathcal{P}_\rho)^{\nu\beta}(q) \right\} i g_{\rho\pi\pi}(p_1 - p_2)_\beta \ . \quad (8)\end{aligned}$$

Here G_F is the Fermi constant, V_{ub} is the relevant CKM matrix element, $g_{\rho\pi\pi}$ is the $\rho\pi\pi$ coupling, l^μ is the leptonic current and $q^2 \equiv (p_1+p_2)^2$ is the squared invariant mass of the 2π system. The hadronic weak matrix element is given by (since we neglect the lepton masses we drop the terms proportional to $(p+p')_\mu$) [9]

$$\mathcal{M}_{\mu\alpha}(B \to V^*) = \frac{2}{\Sigma}\epsilon_{\mu\alpha\rho\sigma} p_B^\rho q^\sigma V(t) + i\{g_{\mu\alpha}\Sigma A_1(t) - \frac{Q_\alpha}{\Sigma}(p_B+q)_\mu A_2(t)\} \quad (9)$$

where $\Sigma \equiv m_B + m_V,\ Q = p_B - q\ (t = Q^2)$ and $V(t),\ A_i(t)$ are Lorentz-invariant form factors. The * symbol means that the vector meson is produced off its mass-shell.

The propagators of the resonances are given by:

$$(\mathcal{P}_i)^{\alpha\beta}(q) = \frac{-i g^{\alpha\beta}}{q^2 - m_i^2 + i m_i \Gamma_i} + (\text{terms in } q^\alpha q^\beta). \quad (10)$$

Since the ρ^I coupling to $\pi^+\pi^-$ is a conserved effective current, *i.e.* $q\cdot(p_1-p_2)=0$, only the transverse component of the vector meson propagators give a non-zero contribution. In addition, because the intermediate ρ^I and ω^I mesons are produced from the recombination of the daughter $\bar{u}$ (in the $\bar{b} \to \bar{u}$ transition) and the spectator u quarks, the hadronic weak amplitudes are related by $\mathcal{M}_{\mu\alpha}(B^+ \to \omega^I) = \mathcal{M}_{\mu\alpha}(B^+ \to \rho^I)$. Thus, Eq. (8) can be written as:

$$\begin{aligned}\mathcal{M}(B^+ \to 2\pi l\nu) \;=\; & i\frac{G_F V_{ub}}{\sqrt{2}} l^\mu \mathcal{M}_{\mu\alpha}(B^+ \to \rho^{I*}) \cdot \frac{g^{\alpha\beta}}{q^2 - m_\rho^2 + i m_\rho \Gamma_\rho} \\ & \times \left\{ 1 + \frac{m^2_{\rho\omega}}{q^2 - m_\omega^2 + i m_\omega \Gamma_\omega} \right\} \cdot i g_{\rho\pi\pi}(p_1 - p_2)_\beta \ . \quad (11)\end{aligned}$$

A straightforward computation of the 2π invariant mass distribution leads to

$$\frac{d\Gamma(B^+ \to 2\pi l\nu)}{dq^2} = \frac{\sqrt{q^2}}{\pi}\frac{\Gamma(B^+ \to \rho^I(q^2)l^+\nu)\cdot\Gamma(\rho^I(q^2)\to\pi^+\pi^-)}{(q^2-m_\rho^2)^2+m_\rho^2\Gamma_\rho^2}\left|1+\frac{m_{\rho\omega}^2}{q^2-m_\omega^2+im_\omega\Gamma_\omega}\right|^2 . \tag{12}$$

The q^2 in the argument of ρ^I means that decay widths must be taken with the ρ^I off its mass-shell.

A very similar evaluation of the 3π mass distribution in the decay $B^+ \to \pi^+\pi^-\pi^0 l^+\nu_l$ gives (in this case $q^2 = (p_1+p_2+p_3)^2$ corresponds to the 3π invariant mass):

$$\frac{d\Gamma(B^+ \to 3\pi l\nu)}{dq^2} = \frac{\sqrt{q^2}}{\pi}\frac{\Gamma(B^+ \to \omega^I(q^2)l^+\nu)\cdot\Gamma(\omega^I(q^2)\to\pi^+\pi^-\pi^0)}{(q^2-m_\omega^2)^2+m_\omega^2\Gamma_\omega^2}\left|1+\frac{m_{\rho\omega}^2}{q^2-m_\rho^2+im_\rho\Gamma_\rho}\right|^2 . \tag{13}$$

The factorization of the decay widths in Eqs. (12) and (13) is an exact result that follows from the conserved effective current conditions in the $\rho \to 2\pi$ and $\omega \to 3\pi$ vertices.

The quasi 'physical' on-shell decay widths of the $B^+ \to \rho l^+\nu$ and $B^+ \to \omega l^+\nu$ decays are obtained by fixing the 2π and 3π invariant masses, respectively, at the ρ and ω meson masses (in practice, the cuts $m_V^2 - \Delta < q^2 < m_V^2 + \Delta$ are necessary to isolate the vector mesons from the q^2 distribution). Under these conditions we get:

$$\left.\frac{d\Gamma(B^+ \to 2\pi l\nu)}{dq^2}\right|_{q^2=m_\rho^2} = \frac{1}{\pi m_\rho\Gamma_\rho}\Gamma(B^+ \to \rho^I l\nu)\cdot B(\rho^I \to 2\pi)|1+\epsilon'|^2, \tag{14}$$

$$\left.\frac{d\Gamma(B^+ \to 3\pi l\nu)}{dq^2}\right|_{q^2=m_\omega^2} = \frac{1}{\pi m_\omega\Gamma_\omega}\Gamma(B^+ \to \omega^I l\nu)\cdot B(\omega^I \to 3\pi)|1+\epsilon''|^2 . \tag{15}$$

Therefore, as already pointed out in Ref. [7], the isospin breaking effects trough ϵ', ϵ'' must be removed from the measured invariant mass distributions quoted in [6] in order to compare quantities related by isospin symmetry. The results given in Eqs. (14) and (15) are identical to the ones obtained in Ref. [7] where it was assumed that the physical quantum mechanical eigenstates for the ρ^0 and ω mesons are given by Eqs. (4)–(5).

Another way to compare the symmetry breaking effects from the prescriptions of Eqs. (1–3) and (4–5) is to decompose the resonant pieces of the amplitudes for 2π and 3π semileptonic B decays. This gives, respectively:

$$\frac{1}{s_\rho}\left\{1+\frac{m_{\rho\omega}^2}{s_\omega}\right\} = \frac{1}{s_\rho}\left\{1+\frac{m_{\rho\omega}^2}{\delta^2}\right\} - \frac{1}{s_\omega}\cdot\frac{m_{\rho\omega}^2}{\delta^2}, \tag{16}$$

$$\frac{1}{s_\omega}\left\{1+\frac{m_{\rho\omega}^2}{s_\rho}\right\} \;=\; \frac{1}{s_\omega}\left\{1-\frac{m_{\rho\omega}^2}{\delta^2}\right\}+\frac{1}{s_\rho}\cdot\frac{m_{\rho\omega}^2}{\delta^2}, \tag{17}$$

where $s_V \equiv q^2 - m_V^2 + im_V\Gamma_V$ and $\delta^2 \equiv m_\rho^2 - m_\omega^2 + i(m_\omega\Gamma_\omega - m_\rho\Gamma_\rho) \approx 2\bar{m}\{m_\rho - m_\omega + i(\Gamma_\omega - \Gamma_\rho)/2\}$ and $\bar{m}$ is the average mass of ρ and ω mesons. Note that the first term in the r.h.s. of Eqs. (16)–(17) would correspond to the use of Eqs. (1–3) and give equal strengths for isospin breaking in the $B^+ \to (\rho^0,\ \omega)l^+\nu$ decay rates. However, the second terms in the r.h.s. of Eqs. (16–17) give very different contributions due to the propagation of the ω (ρ) meson in the 2π (3π) channel.

From Eqs. (14)–(15), the effects of isospin breaking in $B^+ \to \rho^0 l^+\nu$ result more important than in the $B^+ \to \omega l^+\nu$ transition (because $|1+\epsilon'| \approx 1.18,\ |1+\epsilon''| \approx 1.0$). This fact is somehow accidental because $m_\omega - m_\rho \approx \Gamma_\omega$ and therefore the real and imaginary parts in ϵ' have almost equal weights. This situation is quite similar in the $K_L - K_S$ system where $m_{K_L} - m_{K_S} \approx (\Gamma_{K_S} - \Gamma_{K_L})/2 \approx \Gamma_{K_S}/2$ and, therefore, there is not an important numerical difference when computing mixing effects in $K_L \to 2\pi$ decays through Eqs. (1)–(3) or (4)–(5). However, the effects are different in CP violating $K_S \to 3\pi$ decays. As is well known (see [4, 10]), the mixing of states accounts for the complex phase ($\approx \pi/4$) in the CP violation parameters $\eta_{+-,00}$ measured in $K_L \to \pi\pi$ decays. According to the equivalent prescription as the one for the $\rho-\omega$ system, Eqs. (4)–(5) would imply that the complex phase in CP-violating parameters of $K_S \to 3\pi$ decays should be almost zero , which is in clear disagreement with the results obtained using the conventional quantum mechanical eigenstates of Eqs. (1)–(3) that predict the same phase as in $K_L \to 2\pi$.

In practice however, it is difficult to test the difference between both approaches as far as $B^+ \to \omega l\nu$ and $K_S \to 3\pi$ are concerned. . On the one hand, CP violation (and therefore the complex phase of $\eta_{+-0,000}$) has not been observed yet in $K_S \to 3\pi$ decays so as to test whether the mixing of unstable states is given by Eqs. (1)–(3) or (4)–(5). A similar unfortunate situation is present in the $\rho-\omega$ system because the very narrow width of the ω meson does not allow to show up the interference effects due to $\rho-\omega$ mixing by a fine scanning of the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section in the $\rho-\omega$ region as done in $e^+e^- \to \pi^+\pi^-$ [11].

In conclusion, a consistent treatment of unstable particles as provided by quantum field theory, leads to a different mixing scheme for quasi-physical states of a two-level unstable system than the one obtained from the traditional approach based on a $M - i\Gamma/2$ non-hermitian effective hamiltonian. Symmetry breaking effects in truncated observables as isospin violation in semileptonic $B^+ \to (\rho^0,\ \omega)$ transitions or CP violation in $K_L - K_S$ decays turn out to be very different in both approaches.

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$m_X\Gamma_X)$ which reduces to $D^r \approx 2\bar{m}[m_X - m_Y + i(\Gamma_Y - \Gamma_X)/2] = 2\bar{m}D^{nr}$ when the mass splitting is negligible as in $\rho - \omega$ and $K_L - K_S$ systems ($\bar{m}$ is the average mass of X and Y).

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